Example Consider the functions

$$e^x, \frac{x^2}{1000}, \frac{1}{x^2}, 300\sqrt{x}, \frac{1}{x^{2/3}}, x^{2000}$$

Put these functions in order from smallest to largest (1) If x is a very small positive #.

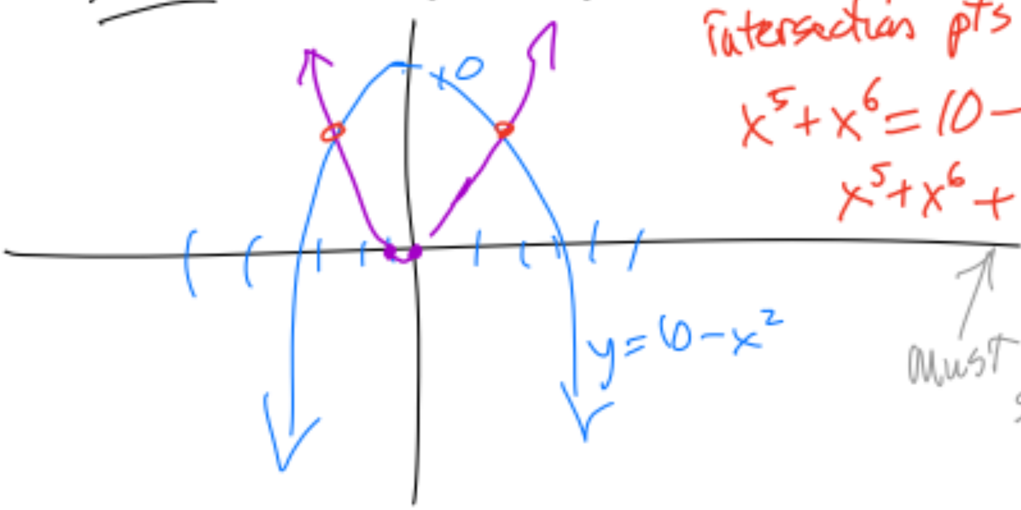
Solution: $x^{2000}, \frac{x^2}{1000}, 300\sqrt{x}, e^x, \frac{1}{x^{2/3}}, \frac{1}{x^2}$

(2) If x is a very large positive #.

Solution: $\frac{1}{x^2}, \frac{1}{x^{2/3}}, 300\sqrt{x}, \frac{x^2}{1000}, x^{2000}, e^x$

Example: Find the area between $y = 10 - x^2$ and $y = x^5 + x^6$

Solution: Where do these graphs intersect?



Intersection pts:

$$x^5 + x^6 = 10 - x^2$$

$$x^5 + x^6 + x^2 - 10 = 0$$

must use software.

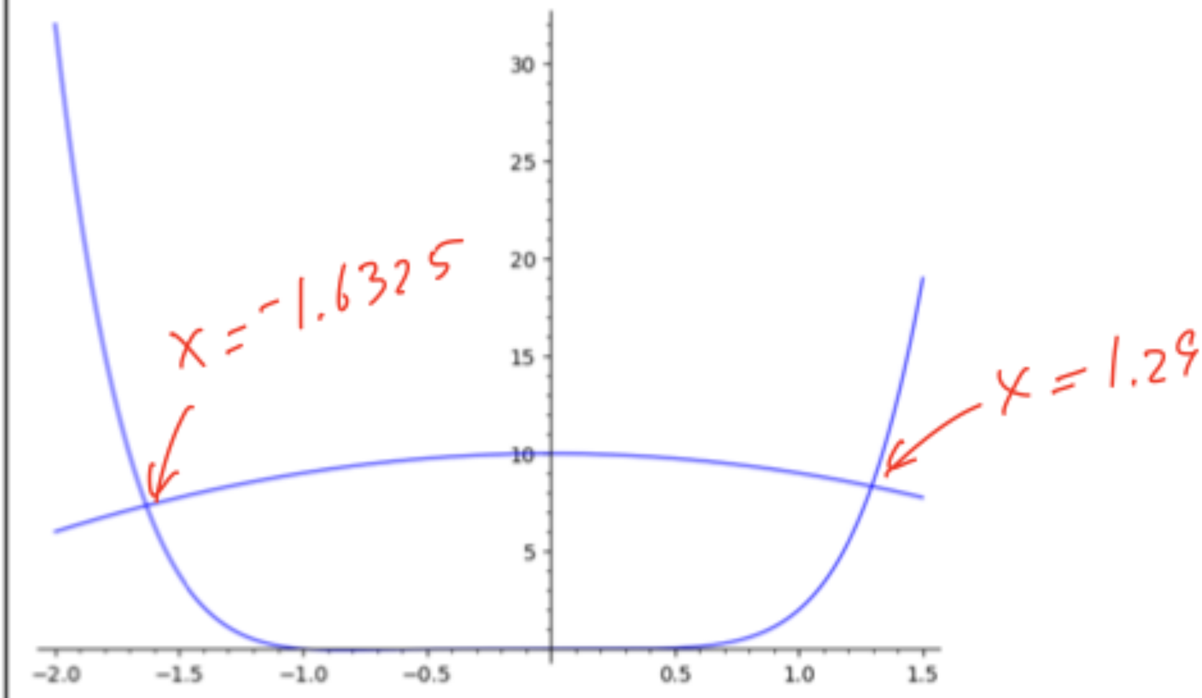
Type some Sage code below and press Evaluate.

```
f(x)=x^5+x^6+x^2-10
a=find_root(f(x),-5,0)
b=find_root(f(x),0,4)
p1=plot(x^5+x^6,(x,-2,1.5))
p2=plot(10-x^2,(x,-2,1.5))
show(a)
show(b)
show(p1+p2)
```

Evaluate

$-1.63253181256414\!08$

1.2940743457196946



$$\text{Area} = \int_{-1.6325}^{1.29} (10-x^2)-(x^5+x^6)\,dx$$

$$= \boxed{24.1826}$$

Example Find the area of the unit ~~circle~~ disk.

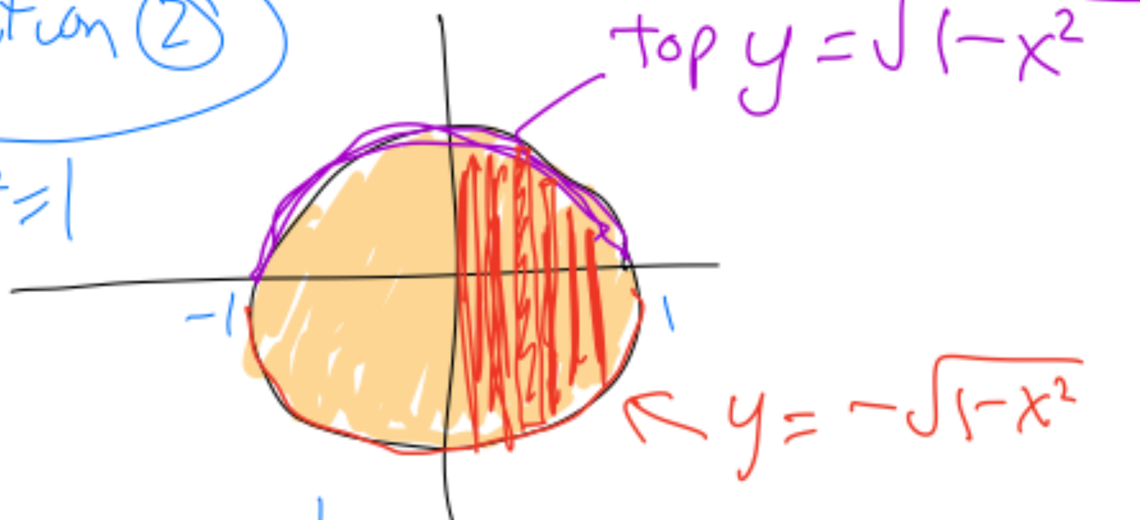
Solution 1

$\pi r^2 = \pi 1^2 = \boxed{\pi}$.

This formula comes from calculus.

Solution 2

$x^2+y^2=1$



$$\text{Area} = \int_{-1}^{1} \left(\sqrt{1-x^2} - \left(-\sqrt{1-x^2}\right)\right) dx$$

$$= \int_{-1}^{1} 2\sqrt{1-x^2}\, dx$$

Let $x = \sin(\theta)$ $dx = \cos\theta\, d\theta$

$\sin\theta = 1$

$$= \int_{-\frac{\pi}{2}}^{\pi/2} 2\underbrace{\sqrt{1-\sin^2\theta}}_{\cos\theta}\,\cos\theta\, d\theta$$

$\sin\theta = -1$

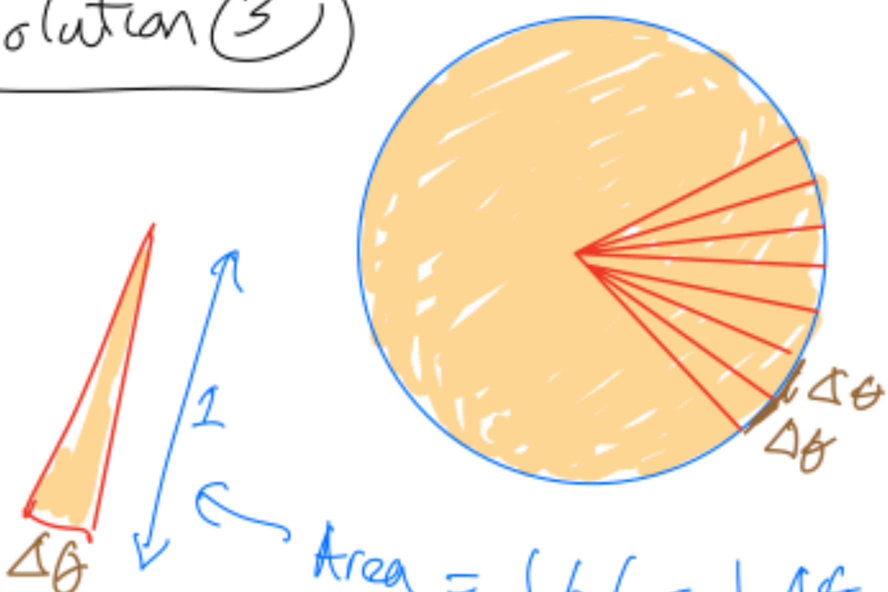
$$= 2\int_{-\pi/2}^{\pi/2} \cos^2\theta \, d\theta = \int_{-\pi/2}^{\pi/2} (1+\cos(2\theta))\, d\theta$$

$$\cos^2\theta = \frac{1}{2} + \frac{1}{2}\cos(2\theta)$$

$$= \theta + \frac{\sin(2\theta)}{2}\Bigg|_{-\pi/2}^{\pi/2} = \frac{\pi}{2} + 0 - \left(\frac{-\pi}{2} + 0\right)$$

$$= \boxed{\pi}.$$

Solution ③



$$\text{Area} = \frac{1}{2} b h = \frac{1}{2}\Delta\theta \cdot 1 = \frac{1}{2}\Delta\theta$$

$$\text{Total area} \approx \sum_{0 \le \theta \le 2\pi} \left(\frac{1}{2}\Delta\theta\right)$$

$$\text{Total area} = \lim_{\Delta\theta \to 0} \left(\text{''} \right)$$

$$= \int_0^{2\pi} \frac{1}{2}\, d\theta = \frac{1}{2}\theta \Big|_0^{2\pi}$$

$$= \frac{1}{2}(2\pi) - \frac{1}{2}(0)$$

$$= \boxed{\pi}.$$

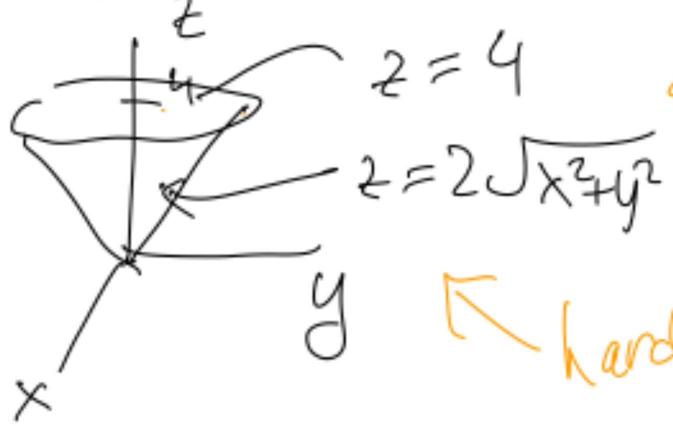
Finding Volumes by Slicing!

Object — sliced up into slices.

Volume $\approx \sum$ Volume of slices.

$$\text{Exact volume} = \int \text{——} \, dx$$

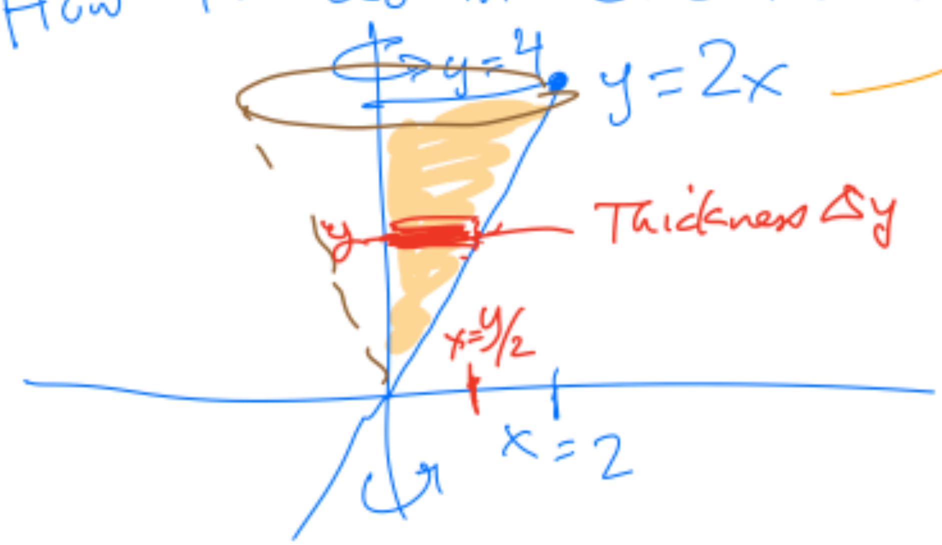
Example: Find the volume of the region between $z = 4$ and $z = 2\sqrt{x^2+y^2}$



hard to see.

$z = 2x$ in $y = 0$ plane

How to do in Calc Two:



slice into horizontal slices.

$$\left.\begin{array}{l}\text{area } \pi x^2 \\ \text{thickness } \Delta y\end{array}\right\} \text{volume} = \pi x^2 \Delta y$$

$$\text{Volume} \approx \sum_{0 \le y \le 4} \pi x^2 \Delta y$$

$$x^2 = \left(\frac{y}{2}\right)^2 = \frac{y^2}{4}$$

limit $\rightarrow$

$$V = \int_0^4 \pi x^2 \, dy$$

$$V = \int_0^4 \pi \frac{y^2}{4} \, dy$$

$$= \pi \frac{y^3}{12} \Big|_0^4 = \frac{\pi \cdot 4^3}{12}$$

$$= \boxed{\frac{16\pi}{3}}.$$

[note v. of cylinder $= \frac{1}{3}\pi r^2 h$]

$= \frac{1}{3}\pi(2)^2 \cdot 4 = \frac{16\pi}{3}$. ✓